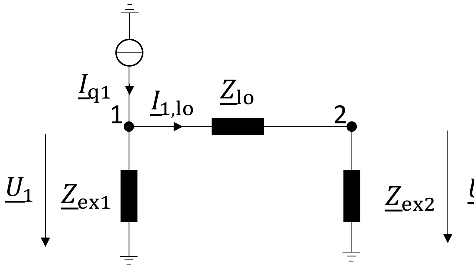
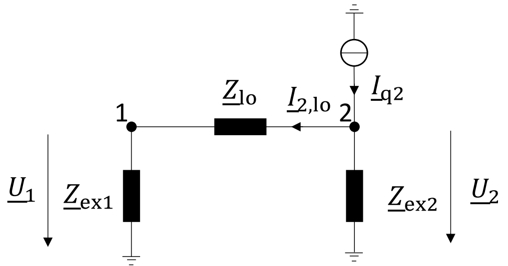
General Definition

To calculate the mutual impedance $\underline{Z}_{12}$, a current is injected to bus 1 and the voltage at bus 2 is calculated, while all other sources in the system are deactivated (voltage sources short-circuited and current sources are open-circuited).

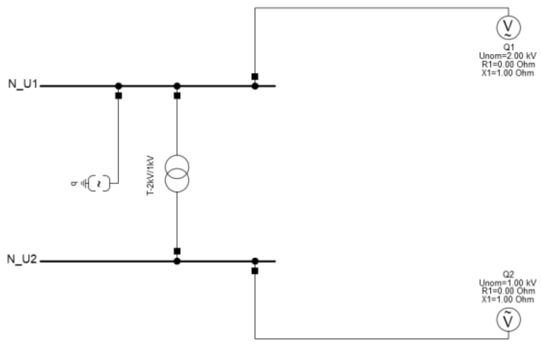


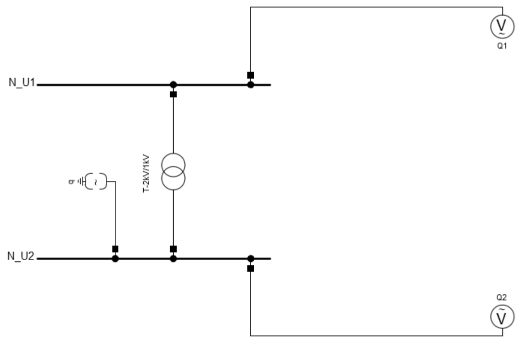


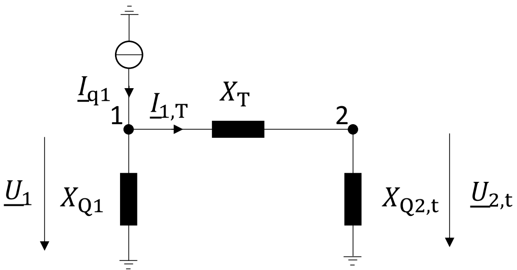
$$\underline{Z}_{21} = \frac{\underline{U}_2}{\underline{I}_{q1}} \text{ and } \underline{Z}_{12} = \frac{\underline{U}_1}{\underline{I}_{q2}}$$

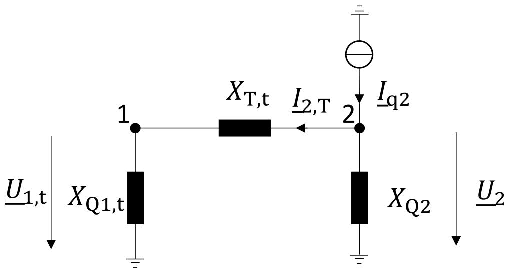
Example Calculation

Checking the example with the Yy5 transformer from the example project, we will show the hand calculation. Please note, due to a better view, all resistance are set to zero.







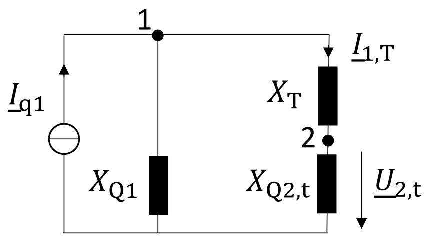


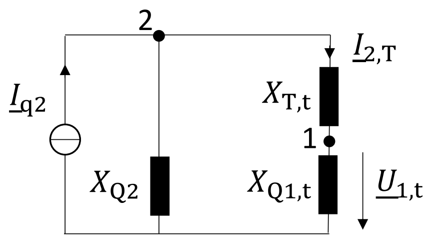
Input Data:

$U_1 = 2$ kV $U_2 = 1$ kV	$x_T = 0.01$ p.u. $S_T = 100$ kVA $\underline{t} = 2 < 150°$	$X_{Q2} = X_{Q1} = 1$ Ohm

Calculation of the transfer values:

$X_T = x_T \frac{(U_1)^2}{S_T} = 0.01 \frac{(2\text{ kV})^2}{100\text{ kVA}} = 0.4$ Ohm $X_{T,t} = x_T \frac{(U_2)^2}{S_T} = 0.01 \frac{(1\text{ kV})^2}{100\text{ kVA}} = 0.1$ Ohm	$X_{Q2,t} = X_{Q2} \cdot \lvert\underline{t}\rvert^2 = 4$ Ohm $X_{Q1,t} = X_{Q1} \cdot \frac{1}{\lvert\underline{t}\rvert^2} = 0.25$ Ohm $\underline{U}_{2,t} = \underline{U}_2 \cdot \underline{t}$, $\underline{U}_{1,t} = \cdot \frac{\underline{U}_1}{\underline{t}}$ / $\underline{I}_{2,T,t} = \underline{I}_2 \cdot \underline{t}^*$, $\underline{I}_{1,t} = \cdot \frac{\underline{I}_1}{\underline{t}^*}$





Current divider

$$\frac{\underline{I}_{1,\mathrm{T}}}{\underline{I}_{\mathrm{q1}}} = \frac{X_{\mathrm{Q1}}}{X_{\mathrm{T}} + X_{\mathrm{Q2,t}} + X_{\mathrm{Q1}}}$$

Current divider

$$\frac{\underline{I}_{2,\mathrm{T}}}{\underline{I}_{\mathrm{q2}}} = \frac{X_{\mathrm{Q2}}}{X_{\mathrm{T,t}} + X_{\mathrm{Q1,t}} + X_{\mathrm{Q2}}}$$

Calculation:

$$\frac{\underline{I}_{1,\mathrm{T}}}{\underline{I}_{\mathrm{q1}}} = \frac{1\ \Omega}{0.4\ \Omega + 4\ \Omega + 1\ \Omega} = 0.185$$

$$\underline{I}_{1,\mathrm{T}} = 0.185 \cdot \underline{I}_{\mathrm{q1}}$$

$$\underline{U}_{2,\mathrm{t}} = \underline{I}_{1,\mathrm{T}} \cdot \mathrm{j}X_{\mathrm{Q2,t}} \rightarrow \underline{U}_2 = \underline{I}_{1,\mathrm{T}} \cdot \mathrm{j}X_{\mathrm{Q2,t}} \cdot \frac{1}{\underline{t}}$$

$$\underline{U}_2 = \underline{I}_{1,\mathrm{T}} \cdot \mathrm{j}(X_{\mathrm{Q2}} \cdot |\underline{t}|^2) \cdot \frac{1}{\underline{t}} = 0.185 \cdot \underline{I}_{\mathrm{q1}} \cdot \mathrm{j}X_{\mathrm{Q2}} \cdot \frac{|\underline{t}|^2}{\underline{t}}$$

$$\frac{\underline{I}_{2,\mathrm{T}}}{\underline{I}_{\mathrm{q2}}} = \frac{1\ \Omega}{0.1\ \Omega + 0.25\ \Omega + 1\ \Omega} = 0.74$$

$$\underline{I}_{2,\mathrm{T}} = 0.74 \cdot \underline{I}_{\mathrm{q2}}$$

$$\underline{U}_{1,\mathrm{t}} = \underline{I}_{2,\mathrm{T}} \cdot \mathrm{j}X_{\mathrm{Q1,t}} \rightarrow \underline{U}_1 = \underline{I}_{2,\mathrm{T}} \cdot \mathrm{j}X_{\mathrm{Q1,t}} \cdot \underline{t}$$

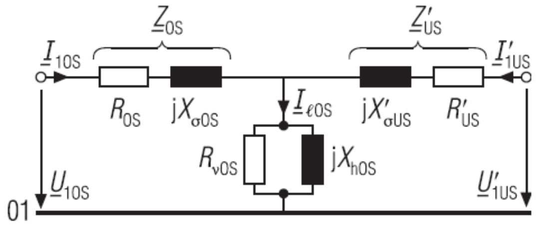
$$\underline{U}_1 = \underline{I}_{2,\mathrm{T}} \cdot \mathrm{j}\left(\frac{X_{\mathrm{Q1}}}{|\underline{t}|^2}\right) \cdot \underline{t} = 0.74 \cdot \underline{I}_{\mathrm{q2}} \cdot \mathrm{j}X_{\mathrm{Q1}} \cdot \frac{\underline{t}}{|\underline{t}|^2}$$

Note: $\underline{t} = 2 < 150°$ and $\underline{t}^* = 2 < -150°$

$$\underline{Z}_{21} = \frac{\underline{U}_2}{\underline{I}_{\mathrm{q1}}} = 0.185\ \cdot \mathrm{j}X_{\mathrm{Q2}} \cdot \frac{|\underline{t}|^2}{\underline{t}} = 0.185\ \cdot \mathrm{j}1\ \Omega \cdot 2 < -150° = 0.37\ \mathrm{Ohm} < -60°$$

$$\underline{Z}_{12} = \frac{\underline{U}_1}{\underline{I}_{\mathrm{q2}}} = 0.74\ \cdot \mathrm{j} \cdot X_{\mathrm{Q1}} \cdot \frac{\cancel{\underline{t}}}{\underline{t}^* \cdot \cancel{\underline{t}}} = 0.74\ \cdot \mathrm{j}1\ \Omega \cdot \frac{1}{2 < -150°} = 0.37\ \mathrm{Ohm} < -120°$$

Alternative point of view: Two-port element



$$\begin{pmatrix} \underline{U}_{1\mathrm{os}} \\ \underline{t} \cdot \underline{U}_{1\mathrm{us}} \end{pmatrix} = \begin{pmatrix} \underline{U}_{1\mathrm{os}} \\ \underline{U}'_{1\mathrm{us}} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_{1\mathrm{os}} \\ \underline{I}'_{1\mathrm{us}} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{\mathrm{h}} + \underline{Z}_{\mathrm{os}} & \underline{Z}_{\mathrm{h}} \\ \underline{Z}_{\mathrm{h}} & \underline{Z}_{\mathrm{h}} + \underline{Z}'_{\mathrm{us}} \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_{1\mathrm{os}} \\ \dfrac{\underline{I}_{1\mathrm{us}}}{\underline{t}^*} \end{pmatrix}$$

$$\begin{pmatrix} \underline{U}_{1\mathrm{os}} \\ \underline{U}_{1\mathrm{us}} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{\mathrm{h}} + \underline{Z}_{\mathrm{os}} & \dfrac{\underline{Z}_{\mathrm{h}}}{\underline{t}^*} \\ \dfrac{\underline{Z}_{\mathrm{h}}}{\underline{t}} & \dfrac{\underline{Z}_{\mathrm{h}} + \underline{Z}'_{\mathrm{us}}}{|\underline{t}|^2} \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_{1\mathrm{os}} \\ \underline{I}_{1\mathrm{us}} \end{pmatrix}$$